MONOCHORD METAPHORS

or

THE POWER OF HARMONIA
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Bhishma Xenotechnites

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AUTHOR’S NOTE

The following remarks were written for and are dedicated to my friend and colleague Lloyd Rodgers, who for several years gave a seminar for music students at the California State University, Fullerton, on the theory and practice of the monochord (and who tells me that his students referred to the instrument as “the god machine”). I regret any and all errors, lapses and misapprehensions I may have committed in what follows, and any resulting distractions. My grateful thanks to Ear Press of Healdsburg, California for the exemplary typesetting and production of this booklet.

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A stretched string is a thing of beauty, and the laws governing its performance are relatively simple.


1. “Tone” comes from Greek *teinein*, “to stretch.”

2. Very few examples in nature of a stretched string; one is spider-web; spiders respond to vibrations (periodic or not) of their web strands.

3. Which was first — musical bow or shooting bow? origins obscure, controversial (see end-note on Homer and *Genji*).

4. On Musical Bow see also Ground Harp (*New Grove*).

5. Monochord may be the first scientific instrument (in modern sense), except for the gnomon (a pole registering time of day, solstice and equinox). Very old: for Greek use, see David Creese’s book. The heliochronometer was a
stretched string whose shadow, like that of a sundial, was accurate enough to be used by the French railways in the 19th century to synchronize its watches and clocks.

6. Monochord direct precursor of many kinds of zither, as well as clavichord (cf. “manicordo” and similar names) (see Arnaud von (de) Zwolle’s invention).


8. “*musica est exercitium arithmaticae occultum nescientis se numer-are animi*” (“music is the hidden arithmetical exercise of a mind unconscious that it is calculating”): Gottfried W. Leibniz, in a letter, quoted in *New Grove*, s.v. (Rudolf Hansa).

9. Number and musical intervals are taken up systematically by Plato in *Timaeus* 34b-36d, there with 2 and 3. He brings in 5 in a complex allegory of social structure in *Republic* 546a-d (v. E.McClain, *Pythagorean Plato*, 174).

10. The monochord is an ideal tool to demonstrate the three principal means and series (given by Archytas in his Fragment 2 (A. Barker, *Greek Musical Writings* 2, 42)). The geometric (ex., 2, 4, 8, 16, etc., or 1/2, 1/4, 1/8, etc.), the arithmetic (ex., 1, 2, 3, 4, etc., or ex., 1, 5, 9, 13, etc.), and
the harmonic (or subcontrary) (ex., 1/2, 1/3, 1/4, 1/5 etc.). The geometric mean of a,b is $\sqrt{ab}$; the arithmetic is $(a+b)/2$; the harmonic is $(2ab)/(a+b)$. The geometric mean of 6 and 12 is $6\sqrt{2}$ ($\sqrt{72}$); the arithmetic is 9; the harmonic is 8 (the latter two of great musical importance to the Greeks, giving the intervals of octave, fourth, fifth and whole tone). On the monochord, the geometric series produces a sequence of identical intervals (octaves, fifths (3/2), 9/4, 27/8, etc.) and so on. The arithmetic series ($m, 2m, 3m, \text{etc.}$) gives the mirror image of the harmonic series (“subharmonics”), which descends. The harmonic series ($m, m/2, m/3, \text{etc.}$) gives the pitches of the ascending harmonic (or “overtone”) series from the fundamental $m$.

11. When stopping the monochord string with the moveable bridge, each new bridge position will give, on either side of the bridge, a new pitch. The number of pitches possible corresponds to the number of points on the line represented by the string, and is unlimited. For any two pitches, however close they are in string-length, a third pitch can be found, theoretically, between them.

12. In producing harmonics on the string by plucking it while touching it at a node (node = “knot”), the string is forced to vibrate in a limited mode; these modes correspond to integral (whole-number) fractions of the string (1/2, 1/3, etc.), and thus the series of harmonics is discontinuous.
13. In traditional number-symbolism (numerology), 1 represents existence (of the string); 2 is generation from the 1, a female principle; 3, the first male principle, gives rise to an unlimited number of new pitches (that are not octave duplications of 1). 2 also gives the matrix (from Gk mētēr, “mother”) of the octave 2:1 into which all possible pitches can be enclosed by (octave) transposition.

14. Mathematics “is given to us in its entirety and does not change, unlike the Milky Way. That part of it of which we have a perfect view seems beautiful, suggesting harmony.” (Kurt Gödel, quoted in Palle Yourgrau, A World Without Time (Basic Books, 2005), 184.)

15. As to the classic question, whether numbers were invented or discovered, the monochord seems to give the latter alternative quite unequivocally, demonstrating the principle of whole numbers as an absolute phenomenon, both visibly and audibly.

16. In the realm of physics and mechanics, the periodic (regularly repeated vs. time) movement of a string (vibration, oscillation) models that of a pendulum or the apparent change of width of a regularly revolving sign (Benade, p. 74ff.) — the string’s velocity is represented in the sine wave, and models also quite closely the change in daylight hours from the vernal equinox, say, through the summer
solstice to the autumnal equinox. With a longer, quite slack string, the oscillations can be observed, and even more dramatically with a strobe light. “String” here regularly means “theoretical string,” which has no mass. Real strings, because of their mass, do not behave quite theoretically, and their harmonics, for example, can depart very slightly from their theoretical value. The ideal string is also perfectly flexible; piano strings’ thickness and rigidity cause them to vibrate to some extent like metal bars, which can cause noticeably “out-of-tune” harmonics.

17. Much of the universe seems to be in some sort of periodic oscillation, from galaxies to atoms. Cyclic (regular circular) movement is, as indicated above, modeled by the periodic movement of the string. Some recent experiments suggest that there may exist the faint resonance in the universe of the energy-wave from the “big bang,” which would be like the ringing of a bell or string, or ripples in a pond, at an unimaginably low frequency. It is possible to imagine that all cosmic events, including my writing this, are in some sense part of the vast harmonic spectrum of such a vast original expanding wave of energy. If the oscillations of the cosmos are real, then Robert Fludd’s famous engraving of the Cosmic Monochord is a sort of adumbration. Note, however, that while the Divine Hand from the cloud can change the tension (pitch) of The Cosmic String, the proportions of the intervals seem to be
represented by Fludd on the monochord as fixed and immutable.

18. As mentioned above, the harmonics, pitches produced by touching the string lightly at a given point while plucking it, only sound when the string is touched at a node, a point that is at a whole-number division of the string. A skilled hand can elicit very high harmonics, 16 and higher. Creating a node forces the string to vibrate in a limited mode; touching the string at 1/5 the string-length from one end will allow pitches of 5, 10, 15 etc. times the open-string frequency, but will prohibit 1, 2, 4, 8... as well as 3, 6, 9, 12... pitches. Note that for 5 or 1/5, touching the string at 2/5, 3/5 and 4/5 the length gives the same result as 1/5, showing the string is vibrating “fundamentally” in 5 equal segments. Prime-number divisions of the string (2, 3, 5, 7, 11 etc.) all function this way, shutting off sectioning by other primes (one way of showing that powers of 2, 3, 5 and so on are incommensurable with one another). The privileged modes of whole-number divisions of the strings, as vibrational states, are discontinuous, and pitches “jump” from one harmonic to the next, with similarities to “quantum leaps” of energy, and to the whole-number behavior of electrons in the atoms of the elements (“Law of Simple Multiple Proportions”). The number of nodal points is theoretically unlimited, but smaller, evidently, than the number of “points” on the string-line, which include irrationals etc.
19. As to the physics of the monochord as a functional unit, it is an example of system equilibrium, where the tension of the string is balanced by the resistance to compression and/or twisting of the structure of the instrument, usually a wooden box.

20. The subharmonic (subcontrary) series has already been mentioned as the “reciprocal” of the harmonic series, a descending sequence $1/m$, $2/m$, $3/m$ … parts of the string, where $(1/m : 2/m)$ is an octave, $(2/m : 3/m)$ a fifth, etc. This series is usually thought of as having no acoustical reality: unlike the harmonic series, its pitches and intervals are not generated by a sounding fundamental pitch. But they can be elicited — a few of them anyway — by a physical frequency-divider (source: Ruland, *Expanding Tonal Awareness*, p. 51): touch a piece of fairly stiff paper to a vibrating string or tuning fork, being careful not to damp the vibrations; if held just right one can hear the pitch drop an octave, then (adjusting the paper) a fifth, and perhaps to other “undertones.” It appears that the paper itself vibrates at $1/2$, $1/3$, etc. the initial frequency, because of its inertia.

21. About the so-called “string theory” of modern particle physics, little can be said here except that the monochord string, which vibrates in multiple modes, appears to model, in a limited way, some theoretical string behavior. (See, e.g., B. Greene, *The Fabric of the Cosmos*, p. 356ff and passim.)
22. The monochord maintained both theoretical and practical utility in music for centuries. The division of its string into many parts was used to demonstrate exact or approximate keyboard temperaments from the 15th to the early 19th century. (See the detailed history of the uses of the instrument from antiquity to its obsolescence by Cecil Adkins; a monochord of reasonable size is not sufficiently accurate to give the pitches of, e.g., equal temperament.) It was a convenient means of teaching musical intervals, and probably also their correct intonation. In his *Compendium of Musical Practice*, translating an earlier treatise of Andreas Ornithoparcus (1517), John Dowland writes that the monochord “tels truth, it cannot tell how to lye,” and “was chiefly invented… to show haire-braind false Musitians their errors, and the way of attaining the truth.” (*ut cervicosis, ac falsis musicis, erroris semitam recludat, ac veritatis viam aperiat.* A *Compendium of Musical Practice*… ed. G. Reese, S. Ledbetter (Dover, 1973), 30, 143.) In his extraordinarily informative treatise of 1752 on flute-playing (and many other aspects of musical performance of the time) (transl. by E. Reilly as *On Playing the Flute* (Schirmer, 1966)), J.J. Quantz recommends the use of a monochord to demonstrate correct intervallic intonation (17.8, p. 269): “Every singer and instrumentalist should be required to acquaint himself with it.” (*Es wäre… nöthig, dass nicht nur ein jeder Sänger, sondern auch ein jeder Instrumentist, sich dieselben bekannt machte.*)
23. Direct, non-metaphorical expression on the monochord is seemingly of sparse occurrence (see Adkins, Ch. VII). The tromba marina, a medieval invention, is essentially a long monochord fitted to an upright resonating body, box-like, to be played with a bow like a cello. Its sounding pitches are those of the string fundamental and upper partials, as on the early (valveless) trumpet, from which it derives part of its name. A few composers wrote music for this instrument, usually as part of an orchestra or other ensemble; the best known are Lully and A Scarlatti (see New Grove, “Trumpet marine” (C. Adkins)). The musical score by Howard Shore to the film The Cell (directed by Tarsen Singh; 2000) is performed by an orchestra that includes a monochord (credited). In “Monochordal Interpretations of Propositions in Euclid’s Elements” (Xenharmonikōn 10, ed. D. Wolf, 1987; reprint, Frog Peak Music; p. 1-16), musicologist and pianist Jon Barlow presents a performed interpretation on the monochord, with commentary, of Propositions 7.1, 7.18, 7.12, 8.11, 6.11, 10.9(a), 10.9(d) (in that order) of Euclid’s famous treatise, an approach that has no precursors that I am aware of, and that shows the monochord as a model for certain basic mathematical operations.

(The word “monochord,” incidentally, is quite rare in the ancient Greek writings on music. Instead, we find regularly the name canon (kanōn, “(measuring) rule”). The verb “to monochordize” (monokhordizein) makes an apparent single appearance, in Aristides Quintilians’s third-century C.E.)
or later treatise, *De musica*, which says that Pythagoras is said to have urged his disciples to “work at the monochord,” explaining that the pinnacle of musical excellence is to be achieved intellectually, through numbers, rather than perceptually, through the hearing. (dio kai Pythagoran phasi … monokhordizein tois hetairois parainesai dēlounta hōs tēn akrotēta, tēn en mousikei, noētōs mallon di’ arithmōn, ē aisthētōs di’ akoēs analēpseōn (97.3-8, GMW 2, p. 497).

24. Philosophical implications. The monochord can be investigated by ear, producing musical intervals of different sizes with the movable bridge, and harmonics of a given string-length by touching the string at certain points. Empirical experiments will lead to recognition that certain relationships of string-lengths produce certain intervals — e.g., a 3 to 2 relationship will always give a perfect fifth, and if one makes this proportion slightly larger or smaller, the fifth will sound out of tune. The proportional relationship of two string-lengths we know as a *ratio*, the Latin word that also means reason (the Greeks’ word was *logos*, a word with many other meanings). We apprehend the musical interval between the pitches through the faculty of perception, and we note the relationship of string-lengths associated with that interval through the faculty of reason, which we can expand into principles that qualify under the modern meaning of “scientific.”

Aristotle is generally considered the West’s first investigative scientist; his influence in many disciplines
from zoology and anatomy to sociology and logic is still powerful. His basic method of investigation was, first, observation and perception (\textit{aisthēsis}), and then the determination or explanation by reason (\textit{logos}) of the perceived phenomena. (A third faculty, memory, played an important role in this method.) The Greek philosophers differed on the relative importance of perception and reason; some, like Pythagoras and Parmenides, considered the perceptions unreliable: the latter in his strange and wonderful poem in epic meter (typical of most didactic poetry), of which we have only a few unconnected ancient fragments, says, “do not let habit counsel you... to wield the aimless eye and noise-filled ear and tongue. But use reason to come to a decision...” (tr. R. Waterfield, \textit{The First Philosophers} (Oxford, 2000), p. 59 (Fr. 7: \textit{mēde s’ ethos... biasthō / nōman askopon omma kai ēkhēssan akouēn, / kai glōssan. krinai de logōi}), “ask[ing] us, for the first time as far as we know, to ‘judge by reason’” (Catherine Osborne, \textit{Presocratic Philosophy: A Very Short Introduction} (Oxford, 2004), p. 48-9).

Plato is very much a part of the Parmenidean-Pythagorean line of thought. His pupil Aristotle could not begin his work without perception, but obviously could not reach any conclusions without reason.

Aristotle’s best known pupil was probably Aristoxenus, known now as a theorist of music, but a writer on many subjects. For him, by contrast, in music at least, the judgment of perception was primary: e.g., ‘For the student of music accuracy of perception stands just about first in
order of importance” (tr. A. Barker, GMW 2, p. 190) (Harmonic Elements 33.22f: tōi de musikōi skhedon estin arkhēs ekhousa taxin hē tēs aisthēseōs akribeia). Aristoxenus’s Elements makes no mention of the monochord or kanōn, nor does it represent musical intervals as numerical ratios (of string lengths), but rather defines them in a very modern, linear way, where, for example, octaves are — as we perceive them — all the same width, and the interval of a fourth is defined as 2½ semitones. He considered the musical theories of the Pythagoreans as “irrelevant” (Barker, GMW 2, 125). (Aristotle, by the way, did not, so far as we know, write a work on music. Nor, surprisingly, on mathematics.)

The influential 20th-century philosopher Ludwig Wittgenstein (1889-1951) belonged to a musical Viennese family, and is said to have had some musical ability himself. It seems unlikely he ever encountered a monochord; his early training was in engineering, in an era — the late 19th and early 20th centuries — in which new technologies were being avidly applied in every area, including musical instruments — the metal-frame, high-tension piano, for example, or the elaborate systems of key-mechanisms applied to woodwind instruments — as well as scientific instruments designed to study acoustics and musical sound (cf., for example, Hermann Helmholtz’s On the Sensations of Tone).

Paralleling this celebration of science and technology came important and influential developments in philosophy, the most important of which was probably that of logical
positivism, associated mainly with the so-called Vienna Circle, with which Wittgenstein had for a time an informal connection. Logical positivism holds that knowledge comes only from sensory phenomena, and its principal goal was to provide a solid logical basis for the scientific method, maintaining also that philosophy itself could be considered to be a science (a position rejected by Wittgenstein).

Wittgenstein’s early treatise, the *Tractatus Logico-philosophicus*, was written mainly during his service in the Austrian army during the First World War. It is a systematic elaboration of the relationship of human language to the world. Perhaps the most famous aspect of the treatise was Wittgenstein’s admission that language was incapable of expressing certain (very important) things — he had in mind matters such as esthetics, ethics, and religion — and where words failed he was compelled to remain silent: “There are, indeed, things that cannot be put into words. They make themselves manifest. They are what is mystical” (*Tractatus* 6.522, quoted in A.C. Grayling, *Wittgenstein: A Very Short Introduction* (Oxford, 2001), p. 19).

I would like to think that Wittgenstein might have found in the monochord an ideal instrument for the manifestation of what can only be “shown, not spoken” (words he evidently also used): the meeting of Aristotle’s perception and reason with perfect clarity (audible and visual): the manifestation, indeed, of truth, as the earlier musicians noted — in fact, of absolute truth; and the demonstration of basic principles of the behavior of matter and energy in the
universe, as well as structural principles: the principle, for
example, of “threeness” and “fiveness.” Beyond its
historical utility in the theory and practice of music, the
monochord seems to me above all the philosopher’s
instrument par excellence.

The master of the monochord must have been the
notable scientist and philosopher, the Alexandrian Claudius
Ptolemy (fl. 146-70 CE), a large portion of whose treatise
on music, the *Harmonics* (a word derived from the Greek
*harmozein*, “to fit together”), he devoted to the many uses of
the canon, from the single-string model to 8- and 15-string
expansions. (Here we might mention the ancient Chinese
instrument the *qin* (*ch’in*), whose history goes back more
than three thousand years, which has (normally) seven
strings of equal length, stretching over a resonant box;
pitches are produced by stopping the string against the
fretless table or as harmonics, certain positions of which are
indicated on the table with inlaid studs. The *qin* has a long
reputation as a scholar’s or philosopher’s instrument,
connected with Confucianism (see *New Grove* s.v.).) Ptolemy’s “treatise is a detailed synthesis of Pythagorean
mathematics and empirical musical observation” (Simon
in Aristotle’s terms, “mathematical harmonics and
harmonics based on hearing” (*harmonikē hē te mathēmatikē kai
hē kata tēn akōen*) (*Posterior Analytics* 79a 1-2, GMW2, p. 71).
(English translation of the *Harmonics* by Andrew Barker,
GMW2, ch. 11.)
In the final chapters of the *Harmonics*, Ptolemy expands the concept of harmonics by analogy to human concerns and to the structural and operational principles of the cosmos. As described in David Creese’s splendidly detailed synopsis and interpretation (Creese, Ch. 6, p.349), Ptolemy aims to show that the conclusions of his harmonic arguments are not limited to the realm of music. … It turns out that harmonics is simply the science in which the ratios and proportions which also govern the structures of the human soul and the *kosmos* are apprehended most readily and accurately: … harmonics, it seems, is the key to the universe.

(Another important study of Ptolemy’s treatise is Andrew Barker’s *Scientific Method in Ptolemy’s “Harmonics”* (Cambridge, 2008).)

In order that Ptolemy may have the last word, a brief quotation from his Book 3, Chapter 3 (92.1-9, GMW2, p. 371) will serve as a closing epigram to these pages:

Since it is natural for a person who reflects on these matters [i.e., the science of harmonics] to be immediately filled with wonder — if he wonders also at other things of great beauty — at the extreme rationality of the power of *harmonia* and the way it finds and creates with perfect accuracy the differences between true forms that belong to it, and since it is also natural for him to desire, through some
divine passion, to behold, as it were, the class to which it belongs, and to know with what other things it is linked among those included in this world-order, we shall try, ... so far as it is possible, to investigate this remaining part of the present study in order to display the greatness of this kind of power (tr. Barker, slightly altered).

(Epei d’ akolouthon an eĩē tōi theōrēsanti tauta, to tethaumakenai men euthus, eĩ kai ti heteron tōn kallistōn, tēn harmonikēn dunamin, hōs logikōtatēn, kai meta pasēs akribeias heuriskhousan te kai poiousan tas tōn oikeiōn eidōn diaphorōs, pothein d’ hupo tinas erōtos theiou kai to genas autēs hōsper theasasthai, kai tisin allois sunhēptai tōn en tōide kosmōi katalambanomenōn; peirasometha ... hōs eni malista, prosepiskepsasthai, touto dē to leipon tēi prokeimenēi theōrīai meros, eis parastasin tou tēs toiautēs dunameōs megethou.)
NOTES

[from p. 3} Genji: “tell my man to get his bow and keep on twanging the string as loud as he can. … Genji’s man had been an Imperial Bowman, and making a tremendous din with his bow he strode toward the steward’s lodge crying “Fire, fire” at the top of his voice.” Lady Murasaki, *The Tale of Genji*, tr. by A. Waley; Dover, 2000, p. 65-7

Homer, *Odyssey* 21.405-11: “now resourceful Odysseus, once he had taken up the great bow and looked it all over, as when a man, who will understand the lyre and singing, easily, holding it on either side, pulls the strongly twisted cord of sheep’s gut, so as to slip it over a new [lyre] peg, so without any strain, Odysseus strung the great bow. Then taking it in his right hand he tested the bowstring, which gave him back an excellent sound like the voice of a swallow.” [tr. Lattimore] [slightly altered and corrected]
Adkins, Cecil D. *The Theory and Practice of the Monochord*. Unpubl. Ph.D. dissertation, State University of Iowa, 1963. Brief history from the Greeks, and a chapter on symbolic use of the monochord, but mostly about theory and practice, from treatises, middle ages into the “post-Renaissance” era (mid 18th c., approximately).


